SOME ASPECTS OF DYNAMICS FOR TWO-BODY TETHERED SPACE SYSTEMS

by

SUDHAKAR. N

DEPARTMENT OF AEROSPACE ENGINEERING

INDIAN INSTITUTE OF TECHNOLOGY KANPUR

MAY, 1991

SOME ASPECTS OF DYNAMICS FOR TWO-BODY TETHERED SPACE SYSTEMS

A Thesis Submitted
in Partial Fulfilment of the Requirements
for the Degree of
MASTER OF TECHNOLOGY

by
SUDHAKAR. N

to the
DEPARTMENT OF AEROSPACE ENGINEERING
INDIAN INSTITUTE OF TECHNOLOGY KANPUR
MAY, 1991

CERTIFICATE

This is to certify that the work "Some Aspects of Dynamics for Two-Body Tethered Space Systems", has been carried out under my supervision and has not been submitted elsewhere for a degree.

Krishna Kumar

(Krishna Kumar)
Professor
Aerospace Engg. Deptt.
IIT Kanpur

May, 1991

ACKNOWLEDGEMENTS

I express my deep sense of gratitude to my guide Prof. Krishna Kumar for suggesting the topic, providing the required background and his active guidance and encouragement at every stage of this work.

I am greatly indebted to all my friends, who directly or indirectly helped me in carrying out this work. I am thankful to Mr. A.K. Srivastava for helping me in typing this work. Finally, I would like to thank my friends for making my stay here a memorable experience.

Sudhakar. N

CONTENTS

LIST OF FIGURES

LIST OF TABLES

NOMENCLATURE

a_1	=	Semi-major axis of the satellite before capture maneuver
a_1'	=	Semi-major axis of the satellite after capture maneuver
a_2	=	Semi-major axis of the `dead' body before capture maneuver
a_2'	=	Semi-major axis of the `dead' body after capture maneuver
c_i	=	Constants ; $i = 1, 2, \ldots, 6$
e_1	=	Eccentricity of the satellite before capture maneuver
e_1'	=	Eccentricity of the satellite after capture maneuver
e_2	=	Eccentricity of the `dead' body before capture maneuver
e_2'	=	Eccentricity of the `dead' body after capture maneuver
E	=	Eccentric anomaly
G	=	Universal Gravitation Constant
H	=	Constant ; Eqn (2.3)
$\hat{i}, \hat{j}, \hat{k}$	=	Unit vectors along ξ, η, ζ axes, respectively
k	=	Kinetic energy factor
k_i	=	Constants ; $i = 1, 2, \ldots, 4$
K	=	earth's gravitational constant
l_1	=	Length of the tether between c.m. and m_1
l_2	=	Length of the tether between c.m. and m_2
l_t	=	Length of the tether
m_o	=	Mass of the hook
m_1	=	Mass of the satellite
m_2	=	Mass of the `dead' body
M	=	Mean anomaly
n	=	Angular velocity
O	=	c.m. of the two massive particles (Fig. 4)

p_i = Constants ; $i = 1,2$; Eqns. (2.1) - (2.2)

Q = c.m. of the tethered system (Fig. 4)

$\bar{r}$ = the radius vector of c.m. of the tethered bodies

r_i = the radial distance P_iQ ; $i = 1,2$ (Fig. 4)

r_h = the radial distance of the hook from the earth's centre (Fig. 1)

$\bar{r}_1$ = the radius vector of the satellite from the earth's centre. (Fig. 2)

$\bar{r}_2$ = the radius vector of the 'dead' body from the earth's centre. (Fig. 2)

$\bar{r}_i^j$ = the radius vector P_iQ_j ; $i = 1,2$; $j = 1,2$ (Fig. 4)

t = time

t_i = time of perigee passage ; $i = 1,2$

u_{xi}, u_{yi} = components of the vector $\bar{u}_i$; $i = 1,2$

$\bar{u}_1$ = velocity of the satellite at the time of capture maneuver

$\bar{u}_2$ = velocity of the 'dead' body at the time of capture maneuver

$\hat{u}_i$ = unit vector along $\bar{r}_i$; $i = 1,2$

$\hat{u}_r, \hat{u}_\theta$ = radial and transverse unit vectors, respectively

$\hat{u}_x, \hat{u}_y$ = unit vectors along x, y axes, respectively

$\bar{v}_1$ = velocity of the satellite after capture maneuver

$\bar{v}_2$ = velocity of the 'dead' body after capture maneuver

$\hat{v}_i$ = unit vector along l_i ; $i = 1,2$

v_{xi}, v_{yi} = components of the vector $\bar{v}_i$; $i = 1,2$

α = angular separation between m_2 and c.m. (Fig. 2)

α_i = the angle between $\bar{r}_i$ amd $\hat{v}_1$ (Fig. 4)

β = angular separation between m_1 and c.m. (Fig. 2)

$\delta\bar{r}$ = perturbations in $\bar{r}$, $\delta\bar{r} = \delta\xi\ \hat{i} + \delta\eta\ \hat{j} + \delta\zeta\ \hat{k}$

$= \delta x\ \hat{u}_x + \delta y\ \hat{u}_y + \delta z\ \hat{k}$

ϵ = constant

ϕ = angle between $\bar{r}$ and $\hat{v}_1$ (Fig. 4)

γ_i = direction of $\bar{v}_i$; i = 1,2

λ_i = constant ; i = 1,2 in Eqn (2.2a)

μ = mass of the massive particle P_2

ν_i = mass of tethered bodies ; i = 1,2

ν_e, ν_f, ν_t = constants in Eqn (3.7a), (3.4a), (3.4), respectively

ψ = pitching angle at time 't' (Fig. 2, Fig. 4)

ψ_o = pitching angle at time t = 0

ψ_c = constant

θ = angle between OP and $\bar{r}$ (Fig. 2)

θ_1 = angle between OP and $\bar{r}_1$ (Fig. 1)

θ_2 = angle between OP and $\bar{r}_2$ (Fig. 2)

θ_{2o} = polar angle of the 'dead' space object as measured from the tethered satellite position at t=0, the time of perigee passage

θ_h = angle between OP and hook (Fig. 1)

dot = derivative with respect to time

prime = derivative with respect to the true-anomaly θ

NAME	:	SUDHAKAR. N.
ROLL NO.	:	8910120
THESIS TITLE	:	SOME ASPECTS OF DYNAMICS FOR TWO-BODY TETHERED SPACE SYSTEMS
THESIS SUPERVISOR	:	Prof. KRISHNA KUMAR Dept. of Aerospace Engineering I. I. T., KANPUR

ABSTRACT

Two important dynamic aspects of tethered satellite system (TSS) are considered here. In the first part, the attention is focussed on the extraction of kinetic energy from the fast moving space wandering objects to the relatively slow moving satellites. The study clearly establishes the usefulness of small tethers that enable a significant momentum transfer from space debris to the satellite.

Finally, the dynamics of the two-body TSS with its c.m. at one of the stable Lagrange points namely L_4 or L_5 in the gravitational fields of the two massive bodies is investigated. The results obtained here show that in general the orbital perturbation due to finite tether length causes the c.m. of the tether to slowly drift away from these stable points. The perturbations are only of the second order, however. The orbital drifts over even relatively large life-spans of the TSS turn out to be insignificant.

CHAPTER - 1

INTRODUCTION

1.1 Preliminary Remarks:

The advent of Space Shuttle extended the possibility of exploration and exploitation of the space activities. Tethered Satellite System (TSS) has been conceived as one of the devices to extend the capability of space shuttle to perform scientific/ applications investigations and operational activities. The concept educes a multiuse tethered system with closed-loop control, capable of supporting a payload or satellite suspended from the shuttle cargobay toward or away from the earth and distances upto approximately 100km from the shuttle. These satellite payloads, although tethered to the shuttle, can be placed in their new orbits permanently, retrieved, refurbished and reconfigured for subsequent missions. A detailed description and operation of these has been presented by Rupp and Laue[1].

The successful missions which involved TSS were carried out aboard the Gemini spacecrafts[2]. In these missions, the manned Gemini XI and Gemini XII space vehicles were tethered to the unmanned Agena Vehicle. The former used the rotating configuration that was maintained by Gemini thruster reactor control system and the latter used gravity gradient stabilized configuration.

In 1990 it was proposed to fly first tether deployer/retrieval system in space. But this mission delayed due

to temporary setback to the shuttle programme that followed the Challenger disaster. The first of these missions[3,4], developed and funded by NASA and Italian National Space Plan (CNR) is now proposed for the year 1992-93. It will deploy approximately a 500 Kg subsatellite upward from the shuttle on a 20mm thick kevlar tether of 20km length, where the gravity gradient will stabilize the two masses along a vertical configuration. It will investigate the interaction of tether - which is designed to have a conducting core - with the earth's magnetic and ionospheric environment. A second mission will be flown about two years later, using the same deployer system aboard the shuttle at 220km altitude deploying a subsatellite 100km downward to investigate earth's atmosphere. Several other tether or tether related applications are in incipient stages. The 1990's should see atleast half a dozen such missions which will demonstrate specific tether capabilities and increase our understanding of their benefits and behaviour in space[5].

Umpteen number of possible tether applications have been proposed for a variety of purposes. Some of the applications are briefly summarized below:

1. Electromagnetic Generator:

An insulated conducting tether connected to a space craft and possibly terminated with a subsatellite can generate DC electrical power to supply primary power on board. The DC electrical power is generated by induction through motion of the conducting tether in the geomagnetic field.

2. Space station platform with controlled gravity laboratory:

A linear, three-mass-constellation that consists of the space station and a science/technology platform at opposite ends of a tether in a gravity gradient stabilized configuration is proposed. A platform on an elevator or crawler mechanism would be made to move along the tether length between its two ends for servicing. Alternately, the platform can be used as controlled gravity laboratory wherein the g-level can be regulated by moving the elevator.

3. ULF/ELF/VLF Communications Antenna:

The variations in current through insulated conducting tether connected to a spacecraft and terminated at both the ends with plasma contactors can produce ULF/ELF/VLF waves enabling worldwide communications. This tether antenna can be self-powered using the current induced in it by power or externally powered through an on-board transmitter.

4. Satellite Boost from Orbiter:

A satellite is deployed along the tether line upwards (i.e., away from the earth) from the shuttle orbiter. Momentum exchange takes place from shuttle orbiter to the satellite. Now if the tether is severed, the satellite will move to a higher orbit and at the same time the shuttle will be deboosted somewhat before it is finally brought back to the earth.

5. Tether Rendezvous System:

The tether rendezvous system will be using a `Smart' hook which can capture and retrieve payloads, space shuttle to space station.

1.2 A Brief Review of Literature:

A tether deployed from a space station is attached to a docking module. This module can capture and retrieve the shuttle allowing a remote rendezvous. This shuttle-docking by tether has been discussed by Hunter(6).

A decayed satellite can be either repaired or reboosted into a higher orbit by a rendezvous maneuver with the help of a permanent tether attached to a shuttle. This eliminates the need for launch of a new satellite. The above rendezvous maneuver is also discussed by Hunter(6).

The idea of a tethered satellite approach for attaining low Earth orbit was put forth by Smithsonian Astrophysical Observatory in 1974(7). In their proposal to the Atmospheric, Magnetospheric and Plasmas in Space (AMPS) science working group, they described the use of long tether system called `Skyhook' for low altitude research.

A tether rendezvous system can capture and retrieve payloads, OTV's or space shuttle to space station with the help of a `Smart' hook(8) by a rendezvous maneuver. This tether rendezvous system helps in saving of a great amount of propellant for incoming vehicles.

By using long tethers, transportation of mass from higher or lower orbits can be achieved. The concept of skyhook tether transportation system has been discussed by Penzo[5].

In many-body problem, knowing the initial positions and velocities of three or more massive particles moving under their mutual gravitational forces, one can determine their final positions and velocities at any time. This problem was first formulated by Newton[9]. The particular solutions to the Three-body problem were given by Lagrange in 1772[10]. His circular restricted three-body problem deals with how the two massive particles moving in circular orbits about their common centre of mass affect the motion of the third infinitesimal particle under the influence of their gravitational fields. Poincare, Hill and others[11] made a great deal of study of the circular restricted 3-body problem.

1.3 Purpose and Scope of Investigation :

The investigation proposed here deals with two important aspects related to a tethered satellite system. First, the possibility of using tether for the extraction of kinetic energy (KE) from the fast moving space-wandering 'dead' objects to the relatively slower satellites involved in useful space applications has been studied. Given the initial conditions - namely the positions and velocities of the two bodies - it is proposed to find out the final velocities of the two bodies. This study assumes that the two bodies are moving in the same plane. The satellite is tethered to a small hook through which it captures

the 'dead' spacecraft passing by. The tethered system is assumed to undergo the pitching motion while the roll is ignored. So are the vibrations of the tether having a negligible mass.

Finally, the dynamics of the two-body tethered satellite system with its c.m. at either of the stable Lagrange points namely L_4 or L_5 has been studied. Here, we explore the possibility of using the two-body tethered system at these stable points with a view to exploit the flexibility and versatility it offers.

CHAPTER - 2

EXTRACTION OF KINETIC ENERGY FROM SPACE DEBRIS

Ever since the space age began, the space has witnessed a rapidly escalating build-up of a large number of space objects such as active and spent payloads, rocket bodies and other miscellaneous debris[12]. Eventually, such uncontrolled proliferation of man-made objects is likely to become a major source of hazard to the future space missions. Soon we must address to this problem before it leads to dangerous consequences. The possibility of a space law under the U.N. auspices warranting the space nations to clear one's own debris cannot be ruled out. Here, we focus our attention on a more positive endeavour which aims at extracting significant fraction of kinetic energy of the wandering space objects to boost the lower payloads into higher orbits. In the process, the resulting reduced velocities of the 'dead' bodies may drown them to earth-bound spiralling trajectories leading to their eventual demise through friction heating in the relatively higher density environment.

A direct smooth transfer of energy from one body to another is not feasible in practice. But tether makes this transfer feasible due to its flexibility. Here, the basic concept of such a transfer has been proposed and a simple planar analysis is undertaken to illustrate how it works.

2.1 Problem Description:

Here, we examine the dynamics of a satellite -hook tethered system during the capture maneuver when the hook is made to attach itself to a wandering space object having a significantly higher speed but no practical utility whatsoever. At the end of such a capture maneuver results in stretching and tilting of the tether, which itself is later cut-off. At the end of the sequence of these operations the momentum transfer from the 'dead' body results in a significantly enhanced kinetic energy of the main satellite body. The planar analysis undertaken here mainly aims at estimating the percentage gain in the satellite kinetic energy.

The available data assumed is as follows (Fig.1).

The Satellite-hook tethered system:

Mass = m_1

Semi-major axis = a_1

Eccentricity = e_1

The pitching angle, pitching rate at t = 0 $\Big\}$ = ψ_o , ψ'_o, respectively

Length of the tether = l_t

Distance between c.m. and m_1 = l_1

Distance between c.m. and m_2 = l_2

The orbiting 'dead' body:

Mass = m_2

Semi-major axis = a_2

Eccentricity = e_2

angle of the wandering space object as measured from the d satellite position at t =0, the timeof perigee passage given by θ_{2o}.

We assume that the fast moving 'dead' body approaches the hook of the tethered system, and when sufficiently close, gets attached to it through a suitable capture maneuver that avoids direct hit. Subsequently, the fast moving hook - 'dead' body combination tends to take the satellite along with it causing clockwise pitching of the tethered system and simultaneously transfering a part of the kinetic energy to the satellite in this process.

The details of the geometry of motion and the co-ordinate axes used are shown in Fig. 2. The point '0' is the earth's centre while P represents the orbit perigee. The symbols (r,θ) denote the polar co-ordinates of the c.m. of the tethered system at time 't' when the hook has just captured and combined with the 'dead' body. The corresponding length of the radius vector of the satellite and the attached body are represented by $\bar{r}_1$ and $\bar{r}_2$. The pitch angle is shown as ψ while α and β provide the angular separation between the two masses on either side of the c.m.

The unit vector $\hat{u}_r$ is along the y-axis taken in the direction of local vertical while $\hat{u}_\theta$ is along the x-axis in the transverse direction. The velocity vectors for the two bodies before and after the capture are denoted by the vectors $\bar{u}_1, \bar{u}_2$ and $\bar{v}_1, \bar{v}_2$ respectively while the hook and the tether masses are assumed to be relatively negligible. The symbols u_{xi}, u_{yi} denote the components of the vector $\bar{u}_i$, $i = 1,2$ along x and y axes respectively. Likewise v_{xi}, v_{yi} represent the corresponding components of the velocity vector $\bar{v}_i$, $i = 1,2$.

2.2 Determination of post-capture velocities of masses:

First, the principle of conservation of linear momenta is applied which leads to :

$$m_1\bar{v}_1 + m_2\bar{v}_2 = m_1\bar{u}_1 + m_2\bar{u}_2 = \bar{p} \text{ (say)}$$

Resolving the momentum components along x and y directions,

$$\lambda_1 v_{x1} + \lambda_2 v_{x2} = p_1 \qquad \ldots\ldots\ (2.1)$$

$$\lambda_1 v_{y1} + \lambda_2 v_{y2} = p_2 \qquad \ldots\ldots\ (2.2)$$

where $\lambda_1 = m_1/(m_1+m_2)$; $\lambda_2 = m_2/(m_1+m_2)$ $\qquad \ldots\ldots\ (2.2a)$

Next, we apply the law of conservation of angular momenta of the system about the earth's centre, which leads to

$$\bar{r}_1 \times m_1\bar{v}_1 + \bar{r}_2 \times m_2\bar{v}_2 = \bar{r}_1 \times m_1\bar{u}_1 + \bar{r}_2 \times m_2\bar{u}_2 = \bar{H} \text{ (say)}$$

and hence

$$-\lambda_1 r_1 \cos\beta\, v_{x1} - \lambda_1 r_1 \sin\beta\, v_{y1} - \lambda_2 r_2 \cos\alpha\, v_{x2} + \lambda_2 r_2 \sin\alpha\, v_{y2} = H \qquad \text{........(2.3)}$$

Finally, we use the energy balance. In view of the absence of the direct hit, the loss of kinetic energy during the capture maneuver would be relatively small. The main sources of these losses here are structural damping in the tether and some stressing-unstressing sequence in the vicinity of the points of attachment in the bodies. The actual values of losses would vary depending upon the materials chosen for the tether and the bodies. Here, we take it into account by introducing a factor K defined as follows:

$$k = \frac{\text{KE after the capture maneuver is completed}}{\text{KE before capture maneuver begins}}$$

It is now easy to write the energy balance equation:

$$\lambda_1 (v_{x1}^2 + v_{y1}^2) + \lambda_2 (v_{x2}^2 + v_{y2}^2) = k_1 \qquad \text{....... (2.4)}$$

where $k_1 = k\,[\lambda_1(u_{x1}^2 + u_{y1}^2) + \lambda_2(u_{x2}^2 + u_{y2}^2)]$

Solving the four simultaneous equations from (1) to (4) for the unknowns v_{x1}, v_{y1}, v_{x2}, v_{y2} we obtain the following solution:

$$\left.\begin{aligned}
v_{x1} &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
v_{y1} &= (k_2 v_{x1} - k_4)/k_3 \\
v_{x2} &= (p_1 - \lambda_1 v_{x1})/\lambda_2 \\
v_{y2} &= (p_2 - \lambda_1 v_{y1})/\lambda_2
\end{aligned}\right\} \qquad \text{....... (2.5)}$$

where

$$a = (\lambda_1/\lambda_2)\,[1 + k_2^2/k_3^2]$$

$$b = -\,2[(\lambda_1/\lambda_2)/k_2^3]\;[k_2k_4 + p_1k_3^2 + p_2k_2k_3]$$

$$c = (\lambda_1/\lambda_2)\,[k_4^2/k_3^2] + (p_1^2+p_2^2)/\lambda_2 + 2(\lambda_1/\lambda_2)$$

$$p_2(k_4/k_3) - k_1$$

$$k_2 = \lambda_1 r_2 \cos\alpha - \lambda_1 r_1 \cos\beta$$

$$k_3 = \lambda_1 r_2 \sin\alpha - \lambda_1 r_1 \sin\beta$$

$$k_4 = H + p_1 r_2 \cos\alpha - p_2 r_2 \sin\alpha$$

2.3 Computational Scheme:

The steps followed are given below:

(i) Computation of r, θ, $\dot{r}$, $\dot{\theta}$: For this the following standard Keplerian relations were utilised. For a given time t, the mean anomaly M was obtained using

$$M = n\,(t - t_i)$$

where

$n = \sqrt{K/a^3}$

t_i = time of perigee passage; $t_1 = 0$ for the tethered system and $t_2 \neq 0$ for the 'dead' body.

K = Earth's gravitational constant.

The Kepler's equation

$$M = E - e \sin E$$

was then used iteratively to solve for E to a sufficient accuracy. The values of r and θ for the c.m. of the tethered system are now computed using

$$\tan(\theta/2) = \left[\left(\frac{1+e}{1-e}\right) \right]^{1/2} \tan(E/2)$$

$$\text{and} \quad r = a(1 - e \cos E)$$

These are next utilised to calculate $\dot{r}$ and $\dot{\theta}$ from

$$\dot{r} = e \sqrt{K/[a(1-e^2)]} \sin\theta$$

$$\dot{\theta} = \sqrt{K a(1-e^2)}/r^2$$

Following the above sequence of computations, we thus obtain $\dot{r}$, $r\dot{\theta}$ for the 'dead' body as well as the c.m. of the tethered system. To obtain the velocity components of the satellite, we would be required to take into account the librational motion as well which is therefore examined next.

(ii) Planar Librational motion of the tethered system:

The equation governing the planar librational motion[13] can be written as

$$(1 + e \cos\theta)\psi'' - 2e \sin\theta (1 + \psi') \sin\theta + 3 \sin\psi \cos\psi = 0$$

For small values of ψ, the above equation can be simplified and to first order it can be written as

$$\psi'' + 3 \sin\psi \cos\psi = 2e \sin\theta$$

which has the solution

$$\psi = c_1 \cos(\sqrt{3}\theta) + c_2 \sin(\sqrt{3}\theta) + e \sin\theta$$

$$\psi' = 3[c_2 \cos(\sqrt{3}\theta) - c_1 \sin(\sqrt{3}\theta)] + e \cos\theta$$

For the small amplitude pitching motion we can utilize these approximate analytical results. Otherwise, we would be required to numerically solve the pitching equation for integration. It is now easy to obtain $\dot{\psi}$ as

$$\dot{\psi} = \psi' \cdot \sqrt{KI}/r^2$$

(iii) Computation of r_1, r_h, θ_h. This is accomplished using simple cosine law:

$$r_1 = [r^2 + l_1^2 - 2rl_1 \cos\psi]^{1/2}$$

$$r_h = [r^2 + l_2^2 + 2rl_2 \cos\psi]^{1/2}$$

$$\theta_h = \theta + \sin^{-1}\left\{\frac{l_2 \sin\psi}{r_h}\right\}$$

where r_h, θ_h refer to the position of the hook when the capture maneuver begins.

(iv) Checking the conditions for capture maneuver:

The conditions that must be satisfied at the point where capture maneuver is to be initiated are

$$\theta_2 > \theta_h$$

$$r_2 > r_h \text{ and}$$

$$[r_2^2 + r_h^2 - 2r_2r_h \cos(\theta_2 - \theta_h)]^{1/2} < \epsilon$$

where ϵ is a small length say of the order of a few meteres. It may be pointed out that the point P corresponding to $t = 0$ has to be so chosen that the 'dead' body is then at a location which eventually leads to simultaneously meeting of the above constraints. In the computer program we achieve this by varying the time t_2 , the time of perigee passage for this body and eventually accepting the value of t_2 which satisfies the above constraints simultaneously. Here, in the absence of any specific knowledge of the capture mechanism, we take $\epsilon = 10$ m.

(v) Computation of velocity components u_{x1}, u_{y1}, u_{x2}, u_{y2} :

For this we use the following relations:

$$u_{x1} = r\dot{\theta} - l_1 (\dot{\theta} + \dot{\psi})\cos\psi$$

$$u_{y1} = \dot{r} + l_1 (\dot{\theta} + \dot{\psi})\sin\psi$$

$$u_{x2} = r_2\dot{\theta}_2$$

$$u_{y2} = \dot{r}_2$$

These values are computed for the instant of time at which the capture maneuver occurs.

(vi) Computation of velocity components v_{x1}, v_{y1}, v_{x2}, v_{y2} :

These are obtained using the solutions given by Eqn. (2.5)

(vii) Computation of the orbital changes when the tether is cut-off from the satellite after the capture maneuver.

Here, we use the following relations

$$\Delta e_1 = (e_1' - e_1)$$

$$\% \text{ increase in } a_1 = [(a_1' - a_1)/a_1] \times 100$$

$$\% \text{ gain in KE of satellite} = \left[\frac{v_1^2 - u_1^2}{u_1^2}\right] \times 100$$

$$\Delta e_2 = e_2' - e_2$$

$$\% \text{ decrease in } a_2 = \left[(a_2' - a_2)/a_2\right] \times 100$$

$$\% \text{ loss in KE of the `dead' body} = \left[\frac{v_2^2 - u_2^2}{u_2^2}\right] \times 100$$

where

$$a_i' = (K\, r_i)/(2K - v_i^2 r_i)$$

$$e_i' = \left[\sin^2\gamma_i + \cos^2\gamma_i \left(1 - \frac{r_i\, v_i^2}{K}\right)^2\right]^{1/2}$$

$$i = 1,2$$

$$u_i = \sqrt{u_{xi}^2 + u_{yi}^2} \; .$$

$$v_i = \sqrt{v_{xi}^2 + v_{yi}^2} \; .$$

$$\gamma_i = \tan^{-1}\left[v_{yi} / v_{xi}\right]$$

2.4 RESULTS AND DISCUSSION :

The simulation results are presented in tables. Table.1 shows the effect of tether length on changes/gains in the orbital parameters. Although the use of tether connection is of vital importance for avoiding the direct hit and associated possibility of damage, the smaller tethers would be preferable for larger kinetic energy gains. The corresponding changes in shape and size of the orbit are considerable and also exhibit some dependence on the tether lengths. Besides, the parametric changes and kinetic energy gains are also significantly depend upon the orbital parameters of the space-wandering body [Table.2] as is naturally to be expected.

It may also be of interest to see how the trajectories of the floating debris changes as a result of the above momentum transfer. These results are shown in Table.3. The changes in a and e seem to be quite substantial as also its kinetic energy losses. Depending upon the proposed plan of space mission and knowledge of the various space-wandering objects, it seems feasible to design for a suitable capture maneuver leading to twin benefits with the satellite raised significantly higher altitudes leaving the 'dead ' body behind to incinerate as a result of heating due to friction.

CHAPTER - 3

DYNAMICS AND STABILITY OF TETHERED SPACE SYSTEM AT L_4 AND L_5

The particular solutions for the three-body problem were determined by Lagrange in 1772. He showed that the orbit of every secondary revolving round its primary, has five special fixed points in their relative frame which are called Lagrange points at which the resultant of inertial and gravitational forces is zero. Three of these points denoted by L_1, L_2 and L_3 lie on the line joining the primary and the secondary. The remaining two points called L_4 and L_5 lying on either side of this line form an equilateral triangle with the primary and the secondary (Fig. 3). The points L_4 and L_5 are of special significance being stable. In other words if a small particle with suitable precise initial conditions is kept at either of these two points, it continues to stay in the vicinity of these equilibrium positions when subjected to small disturbances. For certain applications, the use of these points has been proposed in the past and its dynamics has been studied. Here, we propose the possibility of using a two-body tethered system at these Lagrange points with a view to exploit the flexibility and versatility it offers. For an effective application to be feasible, the points L_4, L_5 should be stable with reference to such a tethered system. This section examines the dynamics of a tethered system with its c.m. at the point L_4 and assesses its stability in the small under various parametric conditions.

3.1 Problem Description:

Here, we consider a two-body tethered system with c.m. lying at one of the 'Stable' Lagrange points, namely L_4 or L_5.

The details of the geometry of motion and its co-ordinate axes used are shown in Fig. 4. For convenience special units are chosen, the details of which are as follows:

(i) The distance between two massive particles P_1 and P_2 is taken as unity, i.e., $P_1P_2 = 1$.

(ii) The sum of masses at P_1 and P_2 is taken as unity. Thus if the mass at P_2 is μ, then the mass at P_1 is $1-\mu$. We assume $\mu \leq 1/2$ without loss of generality.

(iii) The unit of time is so chosen that the Universal Gravitational Constant G becomes unity. Let the tether connected masses at Q_1 and Q_2 be denoted respectively by ν_1 and ν_2 which are being treated as infinitesimally small in comparision to μ_1 and μ_2 and which do not affect the motion of the massive bodies at P_1 and P_2. Here, we consider the particular case in which these bodies execute circular motion around the common system c.m. represented by `O'. Hence, we can easily show that

$$OP_1 = \mu_2 = \mu$$

$$OP_2 = \mu_1 = 1-\mu$$

$$n = 1$$

where

n = angular velocity of the massive bodies about the system c.m.

Now, we define an inertial frame ξ-η with ξ passing through O and coinciding with the line P_1P_2 initially i.e., at $t = 0$. The co-ordinate axes x,y denote a rotating frame with x always taken along the line P_1P_2. Then the x-axis would make an angle $\theta = t$ with respect to ξ-axis at time 't'. At this general time, t, the tether length makes an angle ψ with respect to the x-axis. The unit vectors along ξ, η axes are denoted by $\hat{i}$, $\hat{j}$ and that along x, y axes are denoted by $\hat{u}_x$ and $\hat{u}_y$.

The meanings of other symbols used in the figure are as follows:

$\bar{l}_i = \overline{OQ}_i, \quad i = 1,2$

$\bar{r}$ = the radius vector of c.m. of the tethered bodies.

$\bar{r}_i$ = the radius vector P_iQ, $i = 1,2$

$\bar{r}_i^j$ = the radius vector P_iQ_j ; $i = 1,2$; $j = 1,2$

$\hat{u}_i = \bar{r}_i/r_i \; ; \; i = 1,2$

$\hat{v}_i = \bar{l}_i/l_i \; , \; i = 1,2$

α_i = the angle between $\bar{r}_i$ and $\hat{v}_i$; $i = 1,2$

ϕ = the angle between $\bar{r}$ and $\hat{v}_1$

For the massless tether assumed here,

$$\nu_1 l_1 \hat{v}_1 + \nu_2 l_2 \hat{v}_2 = 0$$

$$\nu_1 \bar{l}_1 = - \nu_2 \bar{l}_2$$

$$\hat{v}_2 = - \hat{v}_1$$

3.2 Formulation of Orbital Equations of Motion for TSS :

Let the co-ordinates of Q be (ξ, η, ζ) then the equation of motion of Q is given as follows:

$$(\nu_1 + \nu_2)\,\ddot{\bar{r}} = -\sum_{i=1}^{2} \mu_i \left[\frac{\nu_1\, \bar{r}_i^{\,1}}{(r_i^{\,1})^3} + \frac{\nu_2\, \bar{r}_i^{\,2}}{(r_i^{\,2})^3} \right]$$

After a considerable manipulation, we get

$$\ddot{\bar{r}} = -\sum_{i=1}^{2} \left(\frac{\mu_i r_i}{r_i^{\,3}} \right)^3 - \sum_{i=1}^{2} \left[\mu_i \sum_{j=1}^{2} \left\{ \frac{\nu_j}{(\nu_1+\nu_2)} \left(\frac{\bar{r}_i^{\,j}}{(r_i^{\,j})^3} - \frac{\bar{r}_i}{r_i^{\,3}} \right) \right\} \right]$$

On evaluating the pair of terms in the small paranthesis and neglecting the terms of the order of l_j^3 and smaller, after considerable manipulation we can express the above equation in the simplified form as

$$\ddot{\bar{r}} = -\frac{\mu_1 \bar{r}_1}{r_1^3} - \frac{\mu_2 \bar{r}_2}{r_2^3} + (3/2)\,\nu_f\, l_t^2 \left[\frac{\mu_1}{r_1} \left(1-5\cos^2\alpha_1\right) \frac{\bar{r}_1}{r_1^4} + \frac{\mu_2}{r_2} \left(1-5\cos^2\alpha_2\right) \frac{\bar{r}_2}{r_2^4} + 2\hat{v}_1 \left(\frac{\mu_1}{r_1^4}\cos\alpha_1 + \frac{\mu_2}{r_2^4}\cos\alpha_2 \right) \right] \quad \ldots.\ (3.1)$$

where $\nu_f = \dfrac{\nu_1 \nu_2}{(\nu_1+\nu_2)^2}$

Now for a pure three body system without the tether or when the tether length is made zero, the simplified equation of orbital motion of the particle masses at Q itself becomes

$$\ddot{\bar{r}}_o = -\frac{\mu_1 \bar{r}_1}{r_1^3} - \frac{\mu_2 \bar{r}_2}{r_2^3} \quad \ldots.\ (3.2)$$

The perturbation motion due to the tether length being finite and not zero is therefore obtained from

$$\ddot{\overline{\delta r}} = \ddot{\bar{r}} - \ddot{\bar{r}}_0 = (3/2)\, \nu_f\, l_t^2 \left[\frac{\mu_1}{r_1} \left(1-5\cos^2\alpha_1\right) \frac{\bar{r}_1}{r_1^4} + \frac{\mu_2}{r_2} \left(1-5\cos^2\alpha_2\right) \frac{\bar{r}_2}{r_2^4} + 2\hat{v}_1 \left(\frac{\mu_1}{r_1^4} \cos\alpha_1 + \frac{\mu_2}{r_2^4} \cos\alpha_2 \right) \right] \qquad \ldots . (3.3)$$

Considering the special case when Q is nominally at L_4 or L_5 itself when

$$r_1 = r_2 = 1 \text{ and}$$

$$\alpha_1 = 60^\circ + \psi \; ; \; \alpha_2 = 120^\circ + \psi$$

the above perturbation equation takes the form :

$$\ddot{\overline{\delta r}} = \frac{3}{2} \nu_t \left[\bar{r} - 5 \left\{ \mu_1 \bar{r}_1 \cos^2(60^\circ + \psi) + \mu_2 \bar{r}_2 \cos^2(60^\circ - \psi) \right\} + 2\hat{v}_1 \left\{ \mu_1 \cos(60^\circ + \psi) - \mu_2 \cos(60^\circ - \psi) \right\} \right] \qquad \ldots .(3.4)$$

where,

$$\nu_t = \nu_f l_t^2 \qquad \ldots \quad (3.4a)$$

In order to examine the orbital perturbations, we now need to study the pitching librations of the tethered system which is considered next.

3.3 Equation of Pitching Motion:

On taking the moment about the c.m. of tethered system, i.e., Q, the equation of pitching motion can be written as

$$M_{zz} = I_{zz} (\ddot{\theta} - \ddot{\psi})$$

However as the nominal orbit is assumed to be circular,

$$\ddot{\theta} = 0$$

and hence the librational equation simplifies to

$$I_{zz}\,\ddot{\psi} = -\,M_{zz}$$

where the external moment component M_{zz} about Q can be evaluated as

$$M_{zz} = -\,\bar{l}_1 \times \nu_1 \left[\mu_1 \bar{r}_1{}^1/(r_1{}^1)^3 + \mu_2\, \bar{r}_2{}^1/(r_2{}^1)^3 \right]$$

$$-\,\bar{l}_2 \times \nu_2 \left[\mu_2 \bar{r}_1{}^2/(r_1{}^2)^3 + \mu_2\, \bar{r}_2{}^2/(r_2{}^2)^3 \right]$$

After considerable algebraic manipulation, this expression is obtained in a more useful form when the equation of pitching motion reduces to

$$\ddot{\psi} = \left[-\,(3/2)\nu_1 l_1 l_t/(\nu_1 l_1^2 + \nu_2 l_2^2) \right] \left[r \sin\phi \left\{ (1-2\mu)\cos\psi - \sqrt{3}\,\sin\psi \right\} + (1-\mu)\mu \sin 2\psi \right] \qquad \ldots\ (3.5)$$

For the motion under consideration with the c.m. of the tethered system at L_4 or L_5, the pitching equation can be written in a simplified form as

$$\ddot{\psi} = A_1 \sin 2\psi - A_2 \cos 2\psi \qquad \ldots(3.6)$$

where

$$A_1 = \left[(3/4)\nu_1 l_1 l_t \,/(\nu_1 l_1^2 + \nu_2 l_2^2) \right]$$

$$A_2 = \sqrt{3}\,(1-2\mu) A_1$$

This is a highly nonlinear second order ordinary differential equation and does not admit a known closed-form solution. So for a better appreciation of the system dynamics, we examine the equilibrium positions and their stability in the small.

3.3.1 Stability Analysis of the pitching motion:

First, we obtain the equilibrium configurations by equating the right hand side of Eqn.(3.6) to zero. This leads following equilibrium positions.

$$\psi = \frac{1}{2} \tan^{-1} [\sqrt{3}\ (1-2\mu)]$$

We consider the stability of the equilibrium configurations in some special cases:

Case 1 : $\mu = \mu_1 = \mu_2 = 1/2$.

Here the equilibrium configuration is given by

$$\psi = 0,\ \pi \ \text{or}\ \pm\ \pi/2$$

(a) $\psi = 0$:

Substituting

$\psi = u + 0$ or $u + \pi$

leads to the following pitching equation

$$\ddot{u} = A_1 \sin 2u$$

Linearisation around the equilibrium position reduces it to:

$$\ddot{u} - 2A_1 u = 0$$

This suggests, the system is unstable. Even a small pitching disturbance leads to an exponential pitching growth initially.

(b) $\psi = \pm \pi/2$:

Let us consider the motion around $\psi = \pm \pi/2$

Substituting

$$\psi = u \pm \pi/2$$

and linearising around $u = 0$ reduces the pitching equation to

$$\ddot{u} = -2A_1 u$$

Evidently, the equilibrium configurations $\psi = \pm \pi/2$ are stable. The frequency of persistent non-growing oscillations resulting from an initial distrubance is given by

$$f = (1/2\pi)\sqrt{2A_1} = (1/2\pi)\sqrt{(\tfrac{3}{2}\nu_1 l_1 l_t)/(\nu_1 l_1^2 + \nu_2 l_2^2)} \quad \text{c.p.s.}$$

Case 2 :

$\mu_2 = \mu << 1$. The equilibrium pitching configurations are given by :

$$\psi = (\pi/6 \ , \ 7\pi/6), \ (-\pi/3 \text{ or } 2\pi/3)$$

We now check the stability of each of these equilibrium configurations.

(a) $\psi = \pi/6$ or $7\pi/6$:

Substituting $\psi = u + \pi/6$ or $u + 7\pi/6$

and linearizing around $u = 0$ reduces the pitching equation to

$$\ddot{u} = 4A_1 u$$

which represents unstable pitching motion.

(b) $\psi = -\pi/3$ or $2\pi/3$

Substituting $\psi = u - \pi/3$ or $u + 2\pi/3$ and linearising around $u = 0$, the pitching equation reduces to

$$\ddot{u} = -4A_1 u$$

which implies stable/persistent non-growing oscillations with a frequency given by

$$f = (1/2\pi)\sqrt{4A_1} = (1/2\pi)\sqrt{(3\nu_1 l_1 l_t/(\nu_1 l_1^2 + \nu_2 l_2^2)} \quad \text{c.p.s}$$

3.4 Orbital perturbations of the tethered system :

The orbital perturbations can be significantly influenced by pitching motion. However, in general, for practical applications, the pitch must be controlled to precise values, we now consider some particular cases :

Case 1 $\mu_1 = \mu_2 = \mu = 1/2$, $\psi = \psi_c = \pi/2$

The perturbation equation can then be simplified to

$$\ddot{\delta \bar{r}} = \nu_e \bar{r} \qquad \text{.... (3.7)}$$

where

$$\nu_e = (15/8)\nu_f l_t^2 \qquad \text{.... (3.7a)}$$

and hence its components along the inertial frame can be written as

$$\ddot{\delta\xi} = -(\sqrt{3}\nu_e/2)\ \text{sint}$$

$$\ddot{\delta\eta} = -(\sqrt{3}\nu_e/2)\ \text{cost}$$

$$\ddot{\zeta} = 0$$

Integration of these equationsleads to :

$$\delta\xi = \nu_e [- \sqrt{3}/2 \sin t + c_1 t + c_2]$$

$$\delta\eta = \nu_e [- \sqrt{3}/2 \cos t + c_3 t + c_4]$$

$$\delta\zeta = c_5 t + c_6$$

where c_1, c_2.........c_6 are constants of integration. Transforming the above into the relative frame xyz, the perturbations due to the finite tether length can be written as

$$\delta x = \nu_e \left[(c_1 t + c_2)\cos t + (c_3 t + c_4)\sin t \right]$$

$$\delta y = \nu_e \left[- \sqrt{3}/2 - (c_1 t + c_2)\sin t + (c_3 t + c_4)\cos t \right]$$

$$\delta z = c_5 t + c_6$$

On using the initial disturbance-free perturbation condition

$$x_o = y_o = z_o = \dot{x}_o = \dot{y}_o = \dot{z}_o = 0$$

the orbital perturbations for the undistrubed case are obtained in the final form

$$\delta x = \nu_e \left\{ \sqrt{3}/2 \sin t - \sqrt{3}/2 \; t \cos t \right\} \quad \ldots\ldots \quad (3.8)$$

$$\delta y = \nu_e \left\{ - \sqrt{3}/2 \; (1 - \cos t) + \sqrt{3}/2 \; t \sin t \right\} \quad \ldots\ldots \quad (3.9)$$

$$\delta z = 0 \quad \ldots\ldots \quad (3.10)$$

<u>Case 2</u> $\mu_1 = \mu_2 = \mu = 1/2 \; , \; \psi = 0^0$

The perturbation equation taken the form

$$\delta r = \nu_e \bar{r} \quad \ldots\ldots \quad (3.11)$$

with a new value of ν_e which is now given by

$$\nu_e = - \frac{3}{8} \nu_f \, l_t^2$$

The solution to Eqn.(3.11) can be expressed exactly as shown above provided this new value of ν_e is made use of in the solution. It is easy to consider some degenerate cases when the mass of the secondary becomes negligible i.e. $\mu = \mu_2 = 0$.

3.5 RESULTS AND DISCUSSION :

The above analysis clearly shows the existence of several stable equilibrium pitching configurations for the tethered systems. In the case of primary and secondary bodies having identical masses, the tether configuration with its length

perpendicular to the line joining these two bodies is stable. On the other hand in the situation when mass of the secondary is relatively negligible in comparison to that of the primary, the tethered system aligned with the line joining its c.m. with the primary represents the stable configuration as expected in this reduced case. However, the frequencies of oscillation in these two different cases differ by a factor of $\sqrt{2}$.

In general, the orbital perturbation due to finite tether length causes the c.m. of the tether to drift away from the position of L_4 or L_5. For the practical cases of interest considered where the pitching angle is precisely controlled to some specific value using a suitable control strategy, the perturbations present comprise of periodic as well as secular terms. The parameter ν_e governing the levels of perturbation magnitude is proportional to the square of the tether length.

It is interesting to note that the perturbations are always present. Fortunately, these being only of the second order in dimensionless tether length (i.e. l_t/r), - which itself is extremely small - the orbital departures over a practical life-span of the TSS would be relatively insignificant.

CHAPTER - 4

CLOSING REMARKS

Some important features of the analysis and results based on them are summarized below.

4.1 Kinetic Energy Transfer Analysis:

It is posssible to exploit the kinetic energy of space - wandering-objects for a significant augmentation of velocity through the tether. Normally, small tether lengths are preferable for a more efficient energy transfer. It, however, requires a careful capture maneuver through a suitable hook. The tensions generated and consequent stressing of material of the tether and satellite have not been taken up in the present analysis. The analysis is restricted to in-plane motion. Evidently, there is considerable scope for making this investigations more consummate.

4.2 Tethered Space System Stability at L_4 and L_5 :

The pitching equilibrium configurations for the tethered space system kept at its c.m. at L_4 or L_5 with zero relative velocity have been identified and corresponding perturbation motion determined. In general, second order perturbations cause

the TSS c.m. to continually drift away from the initial position - namely L_4 or L_5. Since the value of the tether length in the dimensionless form is extremely small, the orbital departures even over a relatively large life-spans of the TSS are insignificant.

REFERENCES

1. Charles, C. Rupp and Jay, H. Lane, "Shuttle/Tethered Satellite System", Summary of presentation for the Proceedings of the AAS 1978 Goddard Memorial Symposium, Washington, D.C., 1978, paper 78-048.

2. Lang, D.L., and Nolting, R.K., "Operations with Tethered Space Satellite System", NASA SP-138, Gemini Summary Conference, 1967.

3. Sission, J.M., "Development Status of First Tethered Satellite System", MSFC Report MP-83-034, Jan 1986.

4. Bonifazi, C., "Tethered Satellite (TSS) Core Science Equipment", AAS 86-214 Vol 62, Tethers in space, Ed. P.M., Bainum, Proceedings of Advances in the Astronautical Sciences, 1987.

5. Penzo, Paul. A., "A Low Earth Orbit Skyhook Tether Transportation System", AAS/AIAA, Astrodynamics Specialist Conference, Aug, 1987.

6. Hunter, M.W., "Near Earth Transportation Options and High Payoff Choices", AIAA International Annual Meeting, Baltimore, Maryland, May 1982.

7. Colombo, "Shuttle Borne `Skyhook', A new tool for low Altitude Research", Smithsonian Institution Astrophysical Observatory Report, Sep, 1974.

8. Martin Marietta, "A Study of Transportation Applications of Space Tethers", Final report, MIT Space Systems Laboratory, Contract No. RH4-394007, Feb, 1985.

9. Archie, E. Roy, The Foundations of Astrodynamics, Macmillan, New York, 1965.

10. Danby, J. M.A., Fundamentals of Celestial Mechanics, Macmillan, New York, 1962.

11. Sterne, T.E., An Introduction to Celestial Mechanics, Interscience, New York, 1960.

12. Chobotov, V.A., "The Collision Hazard in Space, "Journal of Astronautical Sciences, July-Sep, 1982.

13. Misra, A.K. and Modi, V.J., "A Survey on the Dynamics and Control of Tethered Satellite Systems", NASA/AIAA/PSN International Conference on Tethers in Space, Arlington, Va., Sep, 1986.

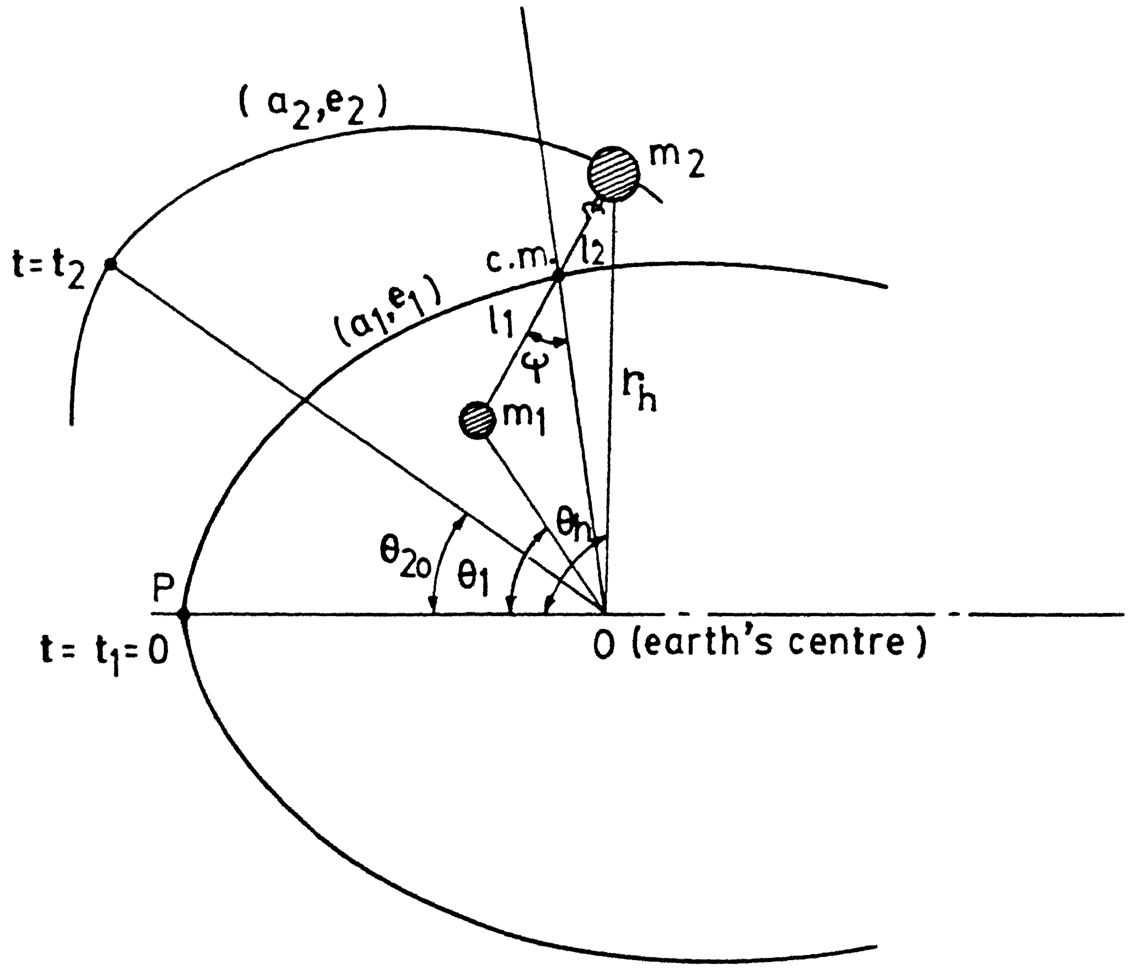

FIG.1 TWO BODY TETHERED SPACE SYSTEM COMBINING WITH THE 'DEAD' BODY

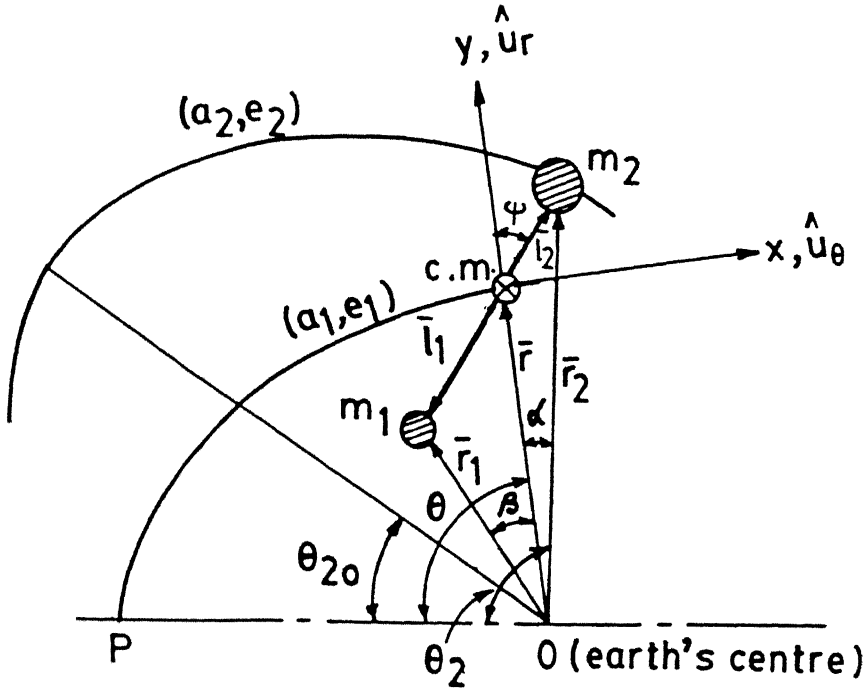

FIG. 2 GEOMETRY OF MOTION OF A TWO BODY TETHERED SPACE SYSTEM AND THE 'DEAD' BODY

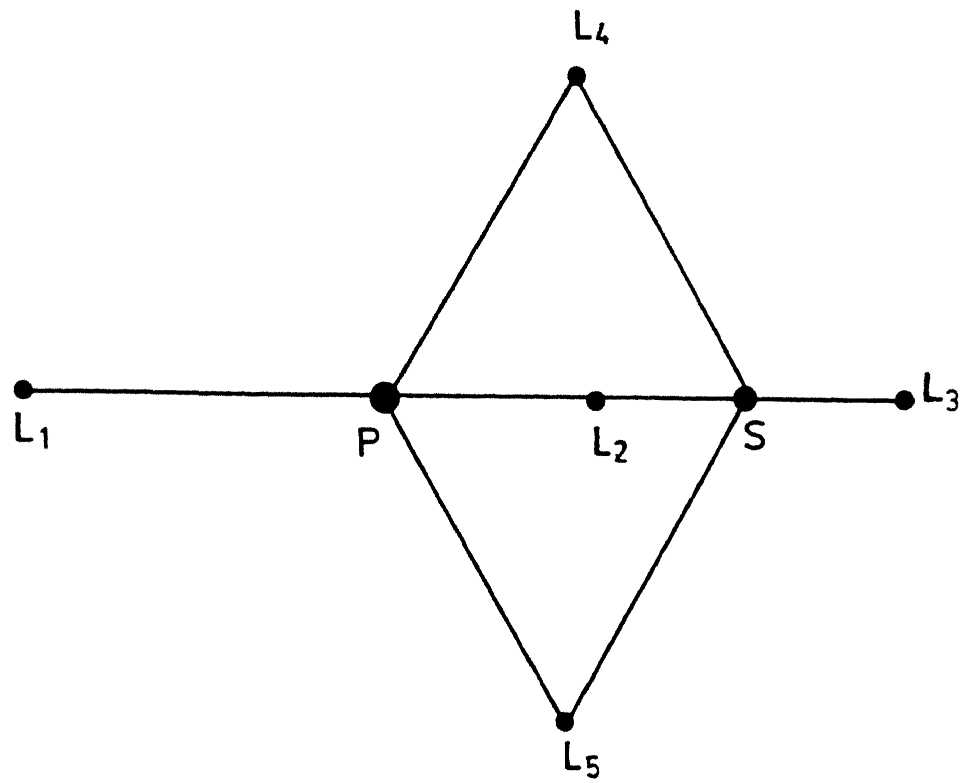

FIG. 3 FIVE LAGRANGE'S POINTS AROUND A PLANET AND ITS SATELLITE

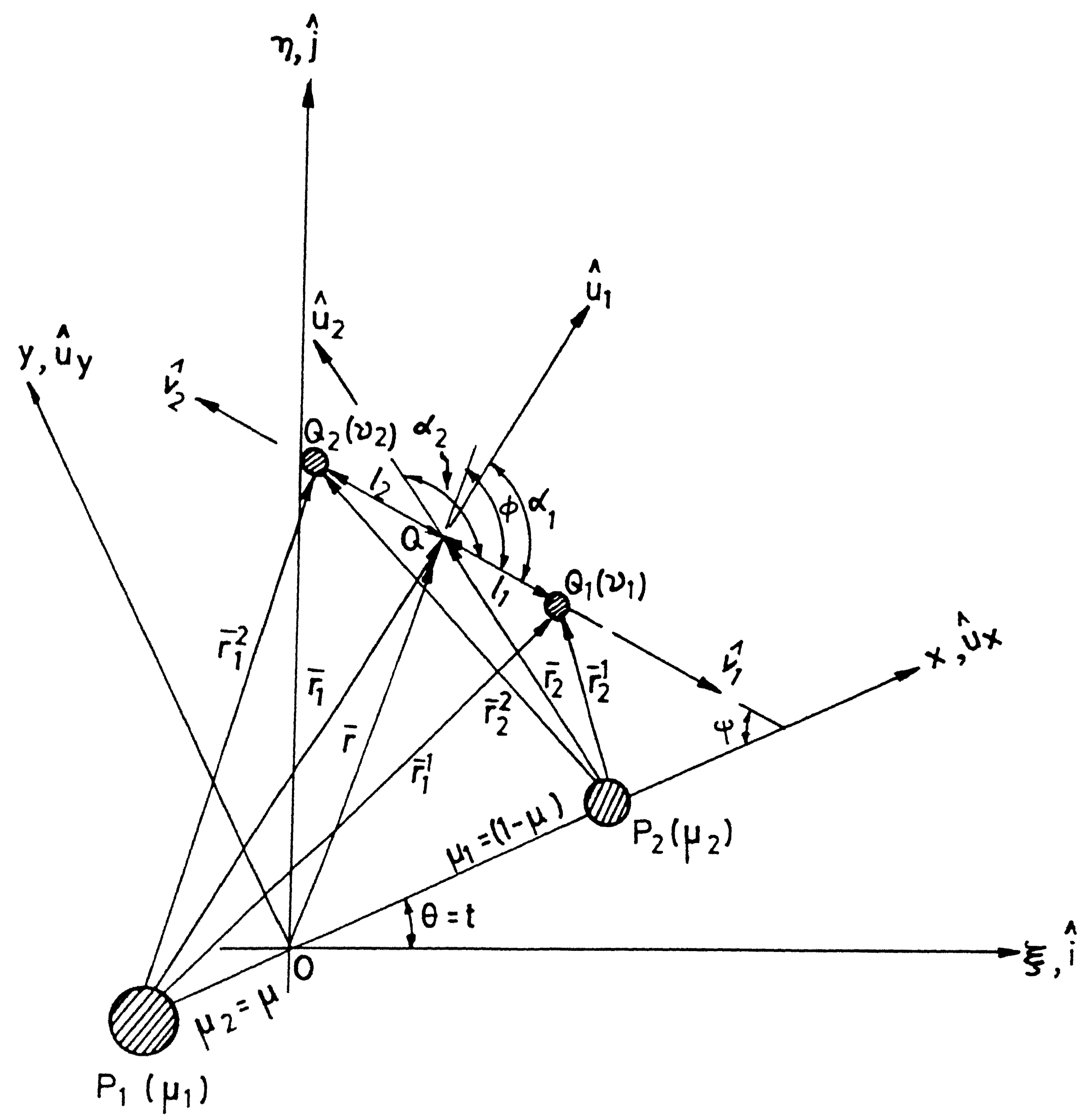

FIG. 4 GEOMETRY OF MOTION OF A TWO-BODY TETHERED SPACE SYSTEM AT A GENERAL POINT AROUND THE PRIMARY AND SECONDARY BODIES

Table 1

Effect of changes in tether length on orbital parameters of the satellite :

$a_1 = 7000$ km , $e_1 = 0.05$, $a_2 = 8000$ km , $e_2 = 0.169$

S.NO.	tether length	change in e_1 (Δe_1)	increase in a_1 (%)	gain in KE (%)
1	1.00000E+02	1.09835E-01	1.28450E+01	1.03227E+01
2	2.00000E+02	9.98915E-02	1.15599E+01	9.40053E+00
3	3.00000E+02	9.57170E-02	1.10889E+01	9.05963E+00
4	4.00000E+02	9.35371E-02	1.08118E+01	8.85754E+00
5	5.00000E+02	9.40565E-02	1.07302E+01	8.80280E+00

Table 2

Effect of changes in the orbital parameters of the space-wandering objects on the satellite orbital parameters :

tether length = 300 m , a_1 = 7000 km , e_1 = 0.05

$m_1 = 10^5$ kg , $m_2 = 10^7$ kg

S.NO.	a_2	e_2	change in e_1 (Δe_1)	increase in a_1 (%)	gain in KE (%)
1	8.00000E+06	1.69000E-01	9.89754E-02	1.13278E+01	9.24490E+00
2	8.10000E+06	1.79000E-01	1.05604E-01	1.18142E+01	9.62015E+00
3	8.20000E+06	1.89000E-01	9.39943E-02	1.05706E+01	8.68642E+00
4	8.30000E+06	1.99000E-01	1.07135E-01	1.18509E+01	9.64536E+00
5	8.40000E+06	2.09000E-01	1.09883E-01	1.20178E+01	9.77202E+00
6	8.50000E+06	2.19000E-01	1.08786E-01	1.14976E+01	9.40191E+00
7	8.60000E+06	2.29000E-01	1.16054E-01	1.236B0E+01	1.00342E+01
8	8.70000E+06	2.39000E-01	1.18073E-01	1.24635E+01	1.01053E+01
9	8.80000E+06	2.49000E-01	1.15547E-01	1.18100E+01	9.63493E+00
10	8.90000E+06	2.59000E-01	1.22217E-01	1.26556E+01	1.02458E+01

Table 3

Effect of changes in tether length on orbital parameters of the space-wandering objects :

$$a_1 = 7000 \text{ km} , \; e_1 = 0.05 , \; a_2 = 8000 \text{ km} , \; e_2 = 0.169$$

S.NO.	tether length	change in e_2	decrease in a_2 (%)	loss in KE (%)
1	1.00000E+02	5.82293E-04	8.02187E-02	9.57515E-02
2	2.00000E+02	2.32965E-04	8.90625E-02	8.73239E-02
3	3.00000E+02	1.45137E-05	8.46875E-02	8.41924E-02
4	4.00000E+02	-5.91427E-04	1.06850E-01	8.22626E-02
5	5.00000E+02	-6.60300E-04	1.07056E-01	8.16756E-02